The Arrow of Time

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Abstract

In this project I discuss and compare past, current and possible future theories on the arrow of time, all of which try to answer why time flows as it does, despite the lack of temporal direction in our scientific laws. The problem ranges from ponds, boxes of gas, and to the beginning (and end) of the universe, as we shall see.

Contents

1 Introduction

When discussing the arrow of time, one must first consider what it is that is being discussed. At a very fundamental level, the word *arrow* implies a direction - we move from *the past* to *the future*, and never vice versa. Is this simply an anthropological accident, or does the entire universe follow this direction? Arthur Stanley Eddington coined the phrase “time’s arrow” in 1927, and in his book *The Nature of the Physical World* states:

> “Let us draw an arrow arbitrarily. If as we follow the arrow we find more and more of the random element in the state of the world, then the arrow is pointing towards the future; if the random element decreases the arrow points towards the past. That is the only distinction known to physics. This follows at once if our fundamental contention is admitted that the introduction of randomness is the only thing which cannot be undone. I shall use the phrase ‘time’s arrow’ to express this one-way property of time which has no analogue in space [1].”

He goes on to explain that the “random element” he refers to is entropy, “a measure of the disorganization of physical systems” [2], and he is considering what is now known as the thermodynamic arrow of time - however, there are other arrows. Indeed, part of the problem of determining the nature of the arrow of time is establishing if there is an arrow which is *the* arrow of time, a so-called *master arrow*, or if there are numerous arrows which are merely *an* arrow of time. One can consider a number of arrow candidates including, but not limited to [3–5]:

- Thermodynamic arrow - the entropy of a closed system will always increase with time.
- Cosmological arrow - the universe is expanding as time increases.
- Radiative arrow - waves (electromagnetic, water, or otherwise) never uniformly converge on a point, but are often seen to uniformly diverge.
- Historical arrow - the “accumulation of information...over time”, for example the “cratering of the Moon...preserves a record of its bombardment by asteroids”. Since “information is the opposite of entropy” it would seem that this historical arrow is at odds with the thermodynamic arrow. However, “the process of fixing information in a record is...an irreversible process that generates entropy, so the entropy of the universe as a whole still rises [5].”
- Biological arrow - we are born and then die, but the reverse is never true, and organisms evolve from simple to more complex. Again, these are information fixing processes and thus generate entropy.
- Psychological arrow - we perceive time as moving from the past to the future, and events move from the unknown into memory. Once more, this is information fixing and generates entropy.

In this project I have concentrated on the former three arrows, since the latter three can be reduced to the thermodynamic arrow as touched on above. Two other possible candidates are the kaon, and the quantum arrow. I will discuss them briefly here, but they do not feature heavily in the rest of the project.

The kaon is an elementary particle that appears to break time-reversal symmetry, in fact it is the only (known) particle that does so. The symmetry of natural laws can be summed up in what is known as the CPT theorem. It consists of three operations: C (charge conjugation), which converts matter into antimatter, P (parity inversion), which flips spacial coordinates to their mirror image, and T (time reversal), which reverses the arrow of time. One outcome of CPT theorem is that if CP symmetry is violated, then T symmetry is also violated. Here we find the relation between the kaon and the arrow of time. The kaon is an unstable particle, and thus will decay. The majority of the time, it decays in to a negative pion, a positron and a neutrino. The exact nature of these particles is unimportant, what matters is CP symmetry is satisfied, and thus it is perfectly possible for a negative pion, a positron and a neutrino to form a kaon. However, roughly 1 in 10^9 decays will result in a positive pion, and electron and an anti-neutrino. CP symmetry is violated, and thus so is T symmetry - *the reverse process is not permitted.* This "arrow" seems to have no relationship to the other arrows, and is somewhat obscure, but it does show that not all physical processes are theoretically time reversible [6].

The quantum arrow comes from the fact that quantum mechanics describes a physical system in terms of its wave function. The system's evolution over time corresponds to the evolution of the wave function, which is deterministic and described by Schrödinger's Equation. However, if a measurement is made on the system then the wave function collapses "discontinuously and indeterminately" - a time asymmetric process which "depends on the past but not the future, in a way which does not seem explicable either as a harmless conventional asymmetry or as of the same origins as the thermodynamic asymmetry [4]." As I have chosen to focus on so-called "classical" arrows, the quantum arrow will not feature in this project.

2 The Thermodynamic Arrow

The classic illustration of the thermodynamic arrow is that of a partitioned box containing a gas. At time $t = 0$ the gas particles are restricted to one side of the box. The partition is removed and the behaviour of the gas for $t > 0$ is observed. The gas will spread throughout the box and, if a long enough period of time is allowed to elapse, the particles of the gas will be spread out uniformly in the whole of the box [5]. The uniform state of the gas is known as the Maxwell distribution, "the distribution of velocities for a gas in thermal equilibrium [4]."

This is an example of the second law of thermodynamics, as formulated by Rudolf Clausis, in action. He described it as (translated in Price [4]) "the universal tendency of entropy to increase" [7] and as mentioned in the introduction, entropy is a measure of disorder, so this means the disorder of the system increases. It is clear that this is the case since when the gas particles are restricted they have been ordered in one side of the box, and as the system evolves this order is lost - some of the particles remain in the half they started in, but some move to the other half.

2.1 Boltzmann's H-theorem

How can we derive this time asymmetric behaviour of the system, the increase of entropy with time, from Newton's time symmetric equations of motion? Ludwig Boltzmann attempted just this in the development of his H-theorem. He defined the following quantity for a system of N particles [8]:

$$H(t) = \int f(\mathbf{x}, \mathbf{v}, t) \ln f(\mathbf{x}, \mathbf{v}, t) d\mathbf{v} \tag{1}$$

Here $\mathbf{x} = (x_1, x_2, x_3)$ is the particle position vector, $\mathbf{v} = (v_1, v_2, v_3)$ is the particle velocity vector, t is of course time, and $f(\mathbf{x}, \mathbf{v}, t)$ is the single particle distribution function "which gives the probability density of finding a particle with the given position and velocity at a certain time [9]."

The derivation of the H-theorem is given in section 6, but for now it suffices to say that H is at a minimum when the gas has the Maxwell distribution, and decreases to this minimum for all other distributions. This implies that the Maxwell distribution is an attractor for the system. When the gas has the Maxwell distribution, the entropy is also at a maximum, so maximum entropy implies minimum H. Boltzmann suggested that in fact $-kH$, where k is Boltzmann's constant (about 1.38×10^{-23} joule/kelvin), was nothing more than the entropy, and in doing so showed that the entropy always increased with time if the system was not yet at maximum entropy [4].

However, consider a system of N particles with velocities $\mathbf{v}_1, ..., \mathbf{v}_N$ at time $t = t_0$, and assume the system is approaching very near to the Maxwell distribution. Now, consider a system identical to the first, the only difference being that the particles have velocities $-\mathbf{v}_1, ..., -\mathbf{v}_N$ at $t = t_0$. The system is now *departing* from the Maxwell distribution, and whereas in the first case $dH/dt < 0$, in the second $dH/d(-t) < 0 \Rightarrow dH/dt > 0$, so H is increasing! This is known as Loschmidt's paradox, or the reversibility paradox - it is a paradox because there is nothing in the laws of mechanics that prohibits a system in which the particles have velocities $-\mathbf{v}_1, ..., -\mathbf{v}_N$, and yet we do not observe systems that depart from the Maxwell distribution and lose entropy [10].

So how did Boltzmann arrive at his time asymmetric H-theorem? He committed what Price calls a "temporal double standard", "an argument which could be used equally well in either temporal direction [applied] selectively, in one direction but not the other [4]." In Boltzmann's derivation of H-theorem he uses *stoßzahlansatz*, "assumption of molecular chaos [4]" - the assumption that the probability of velocities of particles are uncorrelated before a collision, but the same is not true after a collision. This amounts to an assumption of time asymmetry to begin with, so it is no wonder Boltzmann came to the conclusion he did [11].

There is also another issue with H-theorem. Since the theorem is ultimately of a statistical nature, it is only true *in general* and not an absolute truth. H can increase, and in fact Poincaré showed that "any finite mechanical system obeying the laws of classical mechanics will return arbitrarily close to its initial state,... provided we wait long enough." This means that given enough enough time, all our gas particles could return to just one side of the box! H will have decreased but then increased again to its initial value. Again, however, we never actually observe this behaviour. The reason for this is that the recurrence time (that is, the time it takes for the system to return to its initial state) for even a small amount of gas is so large that it is longer than the age of the Universe [10].

For example, suppose we want to know how long we would have to wait for all of the particles to return to one side of the box - the left, say. Assuming the particles are randomly distributed, each particle has exactly $\frac{1}{2}$ probability of being in either side of the box, so the probability that all particles will be in the left side is, for N particles, $\left(\frac{1}{2}\right)^N = \frac{1}{2^N}$. This means that if we assume the particles move every time T, we would have to wait $T \times 2^N$ before returning the initial condition. If we take $N = 10^{23}$ (about one "mole" of particles) then we have to wait $T \times 2^{10^{23}}$ - choose any value of T that you like, reasonable or ludicrously unreasonable such as $2^{-100000}$ seconds, and you would still have to wait far, far longer than the age of the universe.

We have seen that Boltzmann's H-theorem can not satisfactorily explain the thermodynamic arrow of time. Boltzmann himself realised this, and reformulated his ideas in order to link entropy and probability in an effort to avoid a time asymmetric argument.

2.2 Entropy, order, probability and phase space

The argument to link entropy and probability goes as follows. A system can be described in terms of its macrostate or microstate. The macrostate is a description of the general properties of the system, where as the microstate is the exact configuration of particles which make up those properties. In the box example, the macrostate at $t = 0$ is "the gas particles are restricted to one side of the box", where as a microstate would be "particle 1 is at position $\mathbf{q}_1$ with momentum $\mathbf{p}_1$, particle 2 is at position $\mathbf{q}_2$ with momentum $\mathbf{p}_2$..." and so on, describing all of the particles. It is clear that multiple microstates map to a single macrostate. If we assume that all microstates are equally likely and thus occur with equal probability then it must be that some macrostates occur more often then others. If a system has two macrostates, one of which can be arrived at from 99 microstates, and the other from just one microstate, we would expect to find the system in the latter state only 1% of the time.

Returning to the box example, we would expect the macrostate "gas spread out uniformly" to occur more often then "gas concentrated in one area", because there are more microstates that correspond to uniformity than to order. Now we can connect order to probability - an ordered macrostate is less likely than a disordered one, since there are fewer microstates that make up that macrostate, and since entropy is a measure of disorder, high-entropy macrostates are more likely than low-entropy macrostates [4].

Another way to think about this is the phase space of the system. This is a $6N$ dimensional space, where N is the number of particles in the system. This means that for even a simple system such as the box example, the number of dimensions are huge, and a single point in the space represents the entire microstate of the system at a particular point [12]. Consider the general point/microstate $X = (\mathbf{q}_1, ..., \mathbf{q}_N, \mathbf{p}_1, ..., \mathbf{p}_N)$ of N gas particles in a box with phase space Ω. The evolution of the system can be determined by Hamilton's equations $d\mathbf{q}_i/dt = \partial H/\partial \mathbf{p}_i, d\mathbf{p}_i/dt = -\partial H/\partial \mathbf{q}_i$, which give a Hamiltonian function $H(\mathbf{q}_1, ..., \mathbf{q}_N, \mathbf{p}_1, ..., \mathbf{p}_N)$. Every macrostate Γ is a subset of Ω, and every microstate X belongs to a macrostate $\Gamma(X)$, which takes up a volume of phase space $|\Gamma(X)|$. As before, there are more microstates corresponding to uniformity, so disordered macrostates take up a larger volume of phase space - proportional to the probability of that macrostate [13]. Boltzmann related the volume of a macrostate to its entropy with the following equation:

$$S(X) = k \ln |\Gamma(X)| \tag{2}$$

Here $S(X)$ is the the entropy of the macrostate $|\Gamma(X)|$, and k is Boltzmann's constant. Clearly $S(X)$ will be at a maximum for the macrostate taking up the largest volume of phase space (that is, the macrostate with the most microstates), and this is of course the Maxwell distribution, the most disordered macrostate and thus the one with maximum entropy [13].

2.3 Time asymmetry?

Consider a system in a low-entropy macrostate. How does it evolve with time? (Note - the evolution can be in either temporal direction.) The system will move to a new microstate which will in turn determine the new macrostate. The available microstates will be a subset of all of the microstates of the system, as some microstates will be inaccessible from the current one. Using the box example, if the initial microstate describes a particular particle as adjacent to the left-hand wall of the box, a microstate in which that particle is adjacent to the right-hand wall of the box is not instantly accessible and the system would have to go through a number of microstates in order for the particle to reach the right-hand wall. If we assume that given a system with N possible microstates and an initial microstate m_0 with m_I inaccessible microstates:

$$\begin{aligned} P(\text{System moves to an inaccessible microstate}|m_0) &= 0 \\ P(\text{System moves to an accessible microstate}|m_0) &= \frac{1}{(N - m_I)} \end{aligned}$$

then we have "equal probability for all microstates consistent with the initial microstate" m_0 and we can work as before, with all accessible microstates being equally likely, and inaccessible microstates ignored. As we have seen, there are more microstates corresponding to high-entropy macrostates than there are corresponding to low-entropy macrostates, so it is more likely that the system will move to a high-entropy macrostate, and thus entropy will have increased. What will happen next? Boltzmann H-theorem says that "a microstate describing a macrostate other than the most likely one is more likely to change so as to move the system toward the most likely state than away from it". The "most likely" macrostate is the Maxwell distribution, which is also the macrostate of highest entropy, so the system will, given enough time, gain more entropy until it reaches equilibrium [14].

Again, this can also be thought of in terms of phase space. If a system starts in a low-entropy macrostate, then it begins by occupying a small region of the phase space, because the volume of phase space corresponding to a macrostate is determined by adding up all the microstate points it contains. When the system evolves (again, in either temporal direction) it will move from its current region to an adjacent region. The largest regions will be those corresponding to high-entropy macrostates, and thus the system is most likely to move in phase space to a higher entropy region than its current one, since a lower entropy region will be even smaller than the current one. Continuing this logic, the system will most likely move (eventually) into the largest region of phase space, which is of course the macrostate of the Maxwell distribution. Once the system is in this region it is unlikely to leave it since is so vast when compared to the regions of other macrostates. Thus the system will evolve toward the Maxwell distribution, and then stay there. By equation (2), the entropy will increase as the system passes through larger and larger volumes of phase space, until reaching a maximum at the Maxwell distribution [12]. Figure 1 illustrates the evolution of a system in phase space:

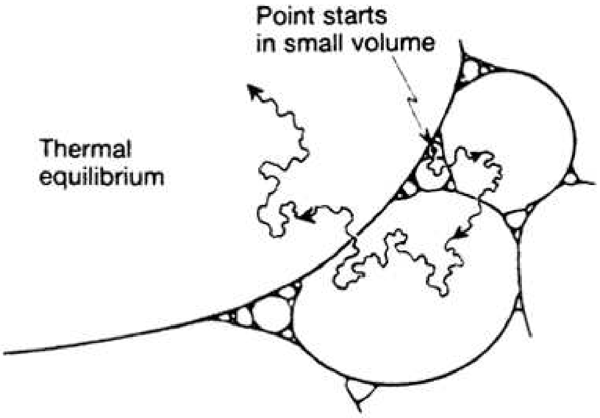


Figure 1: A system's evolution towards equilibrium as it moves through phase space [12]

Where does the time asymmetry come in to this? The answer is clear - it doesn't, since the above arguments work in both temporal directions. In other words, the analysis leads to entropy increasing in *both* directions of time, whereas our real word experience shows that entropy only increases towards what we call the future. Now the problem is not that we have falsely injected a time asymmetry, but the fact that there is none at all! The question has shifted from "why does entropy increase towards the future?" to "why was entropy so low in the past?" [4]. As we shall see in section 3, this is a question which transcends boxes of gas and in fact encompasses the entire universe. The answer is the "Past Hypothesis", which posits that the universe began in the "Past State", the low-entropy initial state of the universe. If the Hypothesis is true then "the most probable history of the universe is one wherein entropy rises [15]."

2.4 Unanswered questions

After this analysis, two questions remain. The first, why is the thermodynamic arrow of time the same for all closed systems? If they are closed then they do not interact with each other, and yet all agree that entropy increases towards the future. As Stenger argues, "you would expect half to have the same arrow as the universe, and half to be opposite [11]." Price calls this "the problem of parallelism" - why do "branch systems" all agree in their temporal alignment? The answer is that if we run time backwards, and ask why "entropy in branch systems always decrease[s] towards the past?" we see that all branch systems "ultimately owe their condition" to the very early universe. Again the question becomes "why was entropy so low in the past [4]?"

The second question is given that high-entropy macrostates are the most likely to occur, how is it possible for any order to arise in the universe at all? For example, the earth and all of the life upon it is far, far from the Maxwell distribution. The reason for this is that the earth is not a closed system, since it constantly receives energy from the sun. This energy is in a relatively low-entropy form as photons of visible light, and is converted to heat energy (that is, infra-red photons) which are then radiated from the earth. This conversion takes place in a number of ways; photosynthesis, burning of fossil fuels, and so on, but the ultimate result is the same. The important thing to consider here is Planck's equation: $E = hv$, in which the energy E of a particle is related to its frequency v by Planck's constant, $h \approx 6.6 \times 10^{-34}$ joule seconds. Since the energy is constant, and the frequency of visible light photons are higher than that of infra-red ones, it must be that there are more infra-red photons being radiated by the earth than there are visible light photons being absorbed. A rough estimate of just how many more can be found by taking the frequency of visible light to be around 600 THz, and that of infra-red to be around 75 THz. Then the energy of one visible light photon is $600/75 = 8$ times more than that of an infra-red photon. In other words for each photo absorbed by the earth, 8 are emitted. Thus, the phase space of the system is far larger, and the entropy increases. Of course, one can then ask how the sun came to have such low entropy, and such a line of questioning leads once more to the early universe [12].

2.5 Summary

Clearly, the entropy of a closed system will, in general, increase towards the future. In this section I have shown that this is the case, however I have also shown that this should hold in the opposite temporal direction, towards the past, as well. In a way, this is to be expected - given that we start with Newton's time-symmetric laws of motion, is it any wonder that there is no asymmetry? It seems that the only way to resolve this is to appeal to initial conditions, to set the entropy in a system to initially be low. Of course, since ultimately (as mentioned above) we are discussing the universe as a whole, we must go beyond thermodynamics to explore the initial conditions of the universe itself.

3 The Cosmological Arrow

As we saw in the previous section, the search for a thermodynamic arrow of time reveals more questions than it does answers. The question of why entropy was so low in the past goes right back to the very early universe, and so we must turn to cosmology for answers - but cosmology also has its own questions. Current observations show the universe to be expanding in all directions, and what is more, the more distant a region of space, the faster it is receding from us. However, this does not mean that our galaxy occupies a central position in the universe, because the view is the same from any galaxy. It also does not mean that there is a "central point" spewing out more universe as expansion continues. Galaxies are like dots on a balloon being inflated - as the balloon gets bigger, each dot moves further away from every other dot, and there is no centre to the surface of the balloon from which more balloon is created [12]. It is this expansion that leads to the concept of a cosmological arrow of time.

3.1 The early universe

The universe is thought to have started 13.7×10^9 years [16] ago with the big bang. This is the origin of the expansion discussed above - the universe began in a gigantic explosion. The radiation from this primordial fireball is in fact still in existence, and is known as black-body background radiation. It has now cooled to a temperature of 2.7 kelvin, and the expansion of the universe has stretched the wavelength into the microwave range. The radiation is found uniformly, throughout the entire universe [12].

The big bang was also responsible for matter (mostly hydrogen) being uniformly distributed in the early universe. As the universe expanded, it cooled and matter formed, and since the expansion was uniform, so too was the distribution of the matter. Based on the previous section on thermodynamics, one might expect that this would mean the universe was in a state of high-entropy. However, unlike a box of gas, the dominant force in the universe is that of gravity. Under the influence of gravity matter forms clumps, and so the most disordered state is one in which all matter is clumped together. Thus the uniformity of the early universe was actually a highly ordered, low-entropy state.

This uniformity was very important in the formation of our current universe, because if it had not been so smooth, galaxies would not have formed. Gravity would have collected matter into much larger clumps, and eventually, "huge black holes". So how did galaxies form? Slight irregularities in the distribution of matter would have been concentrated by gravity over time, and caused it to come together over time, forming the galaxies. So "smoothness is very important" but without some irregularities, the galaxies would not have formed [4].

3.2 The entropy of the universe

How much entropy did the early universe have? One might think that such a question would be impossible to answer, since the big bang lies so far in the past, but actually it is possible using current knowledge to describe the early universe as far back as 10^{-43} seconds after the big bang. This incredibly short amount of time is known as the Planck time and according to current theories, at the Planck time the universe would have been a sphere of radius 10^{-35} metres. This is the Planck length, simply the distance light travels in the Planck time. Below this distance, our current physics break down, as general relativity and quantum mechanics both apply at this level and the two theories are inconsistent [11].

For distances above the Planck length, some conclusions can be drawn. General relativity says that an object in this region of space will be a black hole, and as nothing escapes from a black hole we have no information about it, so it has maximum entropy. The entropy of a black hole radius r is (in Planck units, where Boltzmann's constant $k = 1$):

$$S = (r/r_p)^2 \tag{3}$$

where r_p is the Plank length. For a black hole with radius $r = r_p$, we have $S = 1$, thus the entropy of the universe at the big bang was $S = 1$. Is this the reason that the entropy of the universe was so low, since 1 is not a very large number? The answer is no, because 1 was the maximum entropy possible at that time, due to the very small size of the universe [11].

How does this compare to the entropy of the current universe? Penrose suggests [12] that if the universe consisted entirely of solar-mass black holes (i.e. black holes with the same mass as the sun), the total entropy would be $S = 10^{100}$. This is because "the entropy per baryon in a solar-mass black hole is about 10^{20}", and since there are about 10^{80} baryons in the entire universe (a baryon is a particle made from three quarks, such as the proton or neutron) the total entropy is $10^{20} \times 10^{80} = 10^{100}$. Penrose compares this to the entropy of the background radiation of the universe, about 10^8 per baryon, so the total is $10^8 \times 10^{80} = 10^{88}$ - tiny, compared to the of the black holes.

Of course, a universe made up entirely of black holes is not a very realistic model. Penrose revises [12] his model so that the universe consists of galaxies made up of 10^{11} stars, each with a million solar-mass black hole at the centre. The total entropy per baryon is then

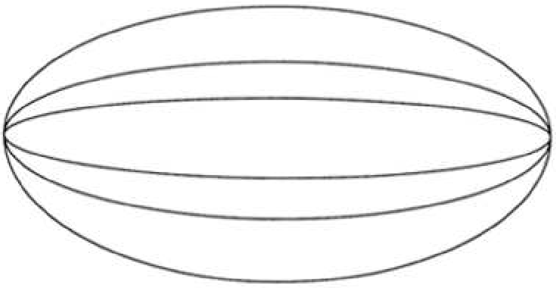
10^{21}, giving a total entropy of 10^{101}. Much time later, when a large part of the galaxies' masses has been drawn into the central black holes, the entropy per baryon will 10^{31}, giving 10^{111}. Taking this forward, and considering a universe undergoing a "big crunch" (a recollapse in on itself, like a big bang in reverse), Penrose considers it "not unreasonable to estimate the entropy of the final crunch...as though the whole universe had formed a black hole." This gives an entropy per baryon of 10^{43}, and a total entropy of 10^{123}.

This figure can be used to give an estimate of the total volume of the phase space of possible universes. Penrose argues [12] "this entropy should represent the logarithm of the volume of the (easily) largest compartment." Recall equation (2); if 10^{123} is the logarithm of the volume, the volume must be $V = e^{10^{123}}$. (Penrose actually gives the value $V = 10^{10^{123}}$, arguing "e and 10 are essentially interchangeable", but I see no reason to do that here.) Similarly, the phase space volume of the current universe is $W = e^{10^{101}}$ or $W = e^{10^{88}}$, depending on if you choose the black hole value or the background radiation value, but either way the value of $W/V \approx e^{-10^{123}}$. This is the probability, in terms of the phase space of all possible universes, of our actual universe occurring.[12] This is an incredibly tiny probability, and yet we (of course!) find ourselves living in the universe that we currently occupy. Some explanation seems to be required.

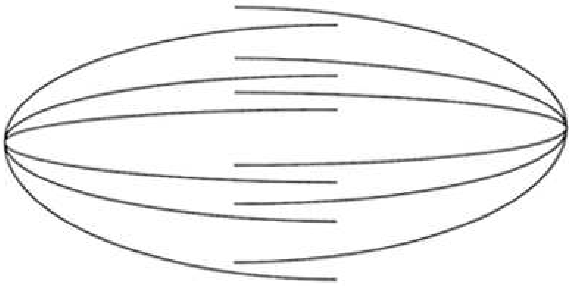
3.3 Possible explanations

Price puts forward four possible explanations for the low-entropy early universe. I shall examine each in turn. The first is that entropy might in fact be low at not just one end of the universe, but both. This view was first put forward by Thomas Gold who suggested that increase in entropy might be connected to the expansion of the universe, in that the expansion might increase the maximum possible entropy, by "creating new possibilities for matter." This would imply that were the universe to contract entropy would decrease as the number of possibilities were reduced. You might think "matter would have to behave in extremely unlikely ways as entropy decreased" towards the future but, as Price points out, it already does towards the past, and so this is not a good argument against a Gold universe. Price claims that there is "little scope...for an explanation of the smoothness of the big bang which does not commit us to the Gold universe [4]."

When considering a Gold universe, there are two possibilities for what might happen when the two low-entropy conditions connect. Price calls these the "meeting" and "mixing" models, as indicated in Figure 2. In the meeting model, objects originate from one singularity, and are then reduced into the other singularity. In other words, the two temporal halves of the universe are a mirror image of each other. This seems to a little ludicrous to me, as one can imagine reaching the "middle" of the universe, and then reversing back along the same course. Price agrees, preferring the mixing model, in which objects originate from either singularity, and evolve towards their future, but not necessarily the other singularity. However I also take issue with this view, as we do not seem to observe anything that might have come from what we would call the future singularity - such as "radiation sinks" in the form of "reverse stars". Perhaps this is due to the fact we are too close to the big bang, and the "mixing" will only occur in our future [4].



The meeting model



The mixing model

Figure 2: Two possible Gold universes[4]

Which ever view you take, the model does of course rely on the fact that the big crunch will actually occur - if it does not, then the maximum possible entropy would continue to increase for all time. However, this is still an open question in cosmology, depending on a number of factors, such as the amount of matter in the universe, and the existence of other more “exotic” materials such as dark matter and dark energy. Such things do not concern us here, because as Price says, “even if the universe as a whole never re collapses...parts of it do”, in the form of black holes. Therefore, “the issue as to whether the universe as a whole recollapses is largely irrelevant to the problem of explaining the low-entropy big bang [4].”

The next explanation was actually suggested by Penrose, following the analysis outlined in the previous section. He supposes there is “an additional asymmetric law of nature”[4], that the Weyl curvature of space-time approaches zero for “initial space-time singularities - but not at final singularities [12].” The Weyl curvature is a component of general relativity, its exact meaning being unimportant in this context, the important idea is that of a time asymmetric law. Also note that the use of the words “initial” and “final” are not a “temporal double standard” [4], because Penrose does not claim that “initial” is correlated with “past”, and “final” with “future”, merely that the Weyl curvature approaches zero at one temporal end of the universe, but not both. This means that from our temporal perspective it approaches zero towards our past and the big bang, but from an (equally valid) reverse perspective, it approaches zero towards our future and the big crunch, since “big bang” and “big crunch” are merely labels for space-time singularities, the choice of which depending on temporal perspective. Taking the big bang to be the initial singularity means that any black holes, as well as the final big crunch, are future singularities in which the Weyl curvature approaches infinity. Penrose goes on to say that to progress any further in our understanding we must know "why space-time singularities have the structures that they appear to have [12]", but as discussed at the start of section 3.2, at distances below the Planck length current physics breaks down. Penrose believes a theory of "quantum gravity" combining the two fields of quantum mechanics and general relativity is what is needed, and that "our sought for quantum gravity is a time-asymmetric theory [12]".

Price classifies the third explanation as a "loophole", originally put forward by Stephen Hawking. The idea is that "a symmetric physical theory might be such that all or most of its possible realizations were individually asymmetric." Price suggests the example of "a factory which produces equal numbers of right-handed and left-handed corkscrews: each corkscrew is spatially asymmetric, but the production as a whole shows no such bias." In other words, perhaps all possible universes are ordered at one end and disordered at the other, and it just so happens that our universe is one in which the ordered end is what we call the big bang, and the disordered what we call the big crunch. Another possible universe, also valid under the "symmetric physical theory" might be such that the arrow of time was reversed, from our perspective [12].

The final explanation is perhaps the most unsatisfactory. It is called "the anthropic approach" [4], and the basic idea is that the universe is the way it is because if it wasn't, we would not be able to exist in it. To question why this should be the case is much like a fish asking why it finds itself in the sea - without water, it could not exist, and without a low-entropy big bang neither could we, or indeed could the fish and all other physical processes that rely on the second law of thermodynamics! Price's objection to this idea is that it requires a much larger reality than the one we observe - a multiverse of universes [4]. As discussed in section 3.2, Penrose estimated that only 1 in $e^{10^{123}}$ universes [12] will have "the right sort of big bang [4]." For this reason, Penrose (and Price) both argue against this explanation. Penrose calculates that "the entire solar system together with all its inhabitants could be created much more 'cheaply'...with an 'improbability' of 'only' one part in much less than $10^{10^{60}}$", which is of course much, much less than Penrose's value of $10^{10^{123}}$ (or $e^{10^{123}}$) [12]. As Price puts it, "the observed universe is vastly more unlikely than intelligent life requires [4]".

3.4 Do we need an explanation?

Another alternative to the explanations in the previous section is that we do not in fact need an explanation. Does a lottery winner demand to know "how?" and "why?" they won despite the incredibly small possibility of them doing so? This is the view that Boltzmann took, saying (translation quoted in Goldstein [13]):

> "The second law of thermodynamics can be proved from the mechanical theory if one assumes that the present state of the universe, or at least that part which surrounds us, started to evolve from an improbable state and is still in a relatively improbable state. This is a reasonable assumption to make, since it enables us to explain the facts of experience, and one should not expect to be able to deduce it from anything more fundamental [17]."

What Boltzmann was suggesting is "in effect an additional physical law [13]." Callender agrees [15], asking "why can't...the Past Hypothesis be a fundamental law?". At first, this seems a little like giving up, and answering "why?" with "because!" However, Callender argues that “virtually everything that happens is unlikely”, and just because the Past State has a low probability, that does not mean that it demands explanation because “explanation and probability [do not] have such a tidy relationship.” He believes that “the Past Hypothesis operates as a fundamental law” and “if one agrees it is a law, then it is...puzzling...to insist that it demands explanation”. His argument for lawhood is that of the Ramsey-Lewis ‘Best System’; “roughly, the laws of nature are the axioms of those true deductive system with the greatest balance of simplicity and strength.” Since the combination of the Past Hypothesis with the second law of thermodynamics allows us to correctly predict increases of entropy in one temporal direction (and one temporal direction alone) Callender states “it seems likely that the Best System would include the Past State as one if its axioms [15].”

However, this seems to me to be no more than a fudge, much like an unscrupulous mathematician evaluating an integral, being displeased with the answer, and so inventing an initial condition to give a constant that makes the answer work! Callender defends his stance by evaluating alternative ways of explaining the Past State, and showing “why none...really count as explaining the Past State.” These are [15]:

- Dynamical explanations: “re-write the dynamics so that the Past State would be generic in the solution space of new dynamics”, that is, replace Boltzmann’s statistical consideration of the second law of thermodynamics in such a way that “the Past State itself emerges” as a likely or natural state - and thus no longer in need of explanation. Of course, this is no easy task.
- Non-dynamical law: Explain the Past Hypothesis with a new law, such as Penrose’s suggestion discussed in section 3.3 that “the Weyl curvature vanishes near the initial singularity and...this...implies low entropy.” Callender’s view is that this “is simply the Past Hypothesis dressed in fancy clothing.” In other words, reformulating the question does nothing towards gaining an answer.
- Eliminating the initial probability distribution: As we saw in section 3.2, the probability of the Past State is incredibly small, but this would not be a problem if physics could be modified to not need an initial probability distribution. Callender indicates the Ghirardi-Rimini-Weber (GRW) interpretation of quantum mechanics as a candidate, as it “would make entropy increase subsequently likely for every nomically possible initial condition.” The Past State would then not be unlikely, because entropy increase will be likely for all initial conditions. However, Callender does not fully explain why this doesn’t “count” for explaining the Past State.
- Anthropic explanations: Callender agrees with both Price and Penorse that this is not a good choice of explanation, as “it posits a substantive - and enormously extravagant - claim about the world,” since as we saw in section 3.3, “the observed universe is vastly more unlikely than intelligent life requires [4]”.

3.5 Summary

In this section I have explored a number of explanations as to why the entropy of the early universe was so low. I have also considered that perhaps an explanation is not in fact needed, and the Past State should stand on its own merit as a law. My feeling is that we should side with Price, in that the Past State does require an explanation. I find Callender’s argument of lawhood to be incomplete, particularly his arguments against the validity of other alternative explanations. I also agree that we should not look towards an anthropic argument for our explanation, for the reasons discussed above. Along these lines I reject the "corkscrew" argument, that we are in an asymmetric universe but all possible universe are collectively symmetric, because again it requires a larger existence that we observe.

This leads to the choice between a Gold universe, in which both ends of the universe have low-entropy, or Penrose's idea of an asymmetric law that distinguishes the big bang from other singularities. I believe that with our current knowledge of the universe, either explanation is a possibility. If we could observe an object from the future singularity, then this would lend support to a Gold universe. Such an observation could be to search for "reverse stars" by looking for areas of the universe which cause a telescope's back plate to cool - because it is "de-absorbing" future radiation [4]. This does seem slightly fantastical, but I don't think entirely impossible. On the other hand, if a future theory of quantum gravity were to successfully include Penrose's Weyl curvature hypothesis, or a similar idea, then the problem would also be solved. Perhaps, however, there is another solution. We have seen the link between the thermodynamic and cosmological arrows of time, but there is a third arrow - the arrow of radiation. Could this arrow provide an explanation for the arrow of time?

4 The Radiative Arrow

Imagine a circular pond, its surface still and undisturbed. A pebble is dropped into the centre of the pond, the water ripples, rings radiating out from the point of impact, and the pebble sinks to the bottom of the pond. Now imagine the same situation but with time reversed. A pebble leaps up off the bottom of the pond, meanwhile water rings converge on the centre of the surface, and as the pebble reaches the surface the rings propel it in to the air. As before, this behaviour in forward time is just as permissible in backwards time, but we never observe pebbles spontaneously launching themselves from ponds. It would seem from our experience that waves only radiate uniformly when diverging from a source, and never when converging to a sink [4]. This would indicate another arrow of time - but how does it relate to the others?

4.1 Thermodynamic explanation

Is it possible to derive the radiative arrow of time from the thermodynamic arrow of time? Popper believes this to be the case, and argues using the above pond example. He says that for the reverse time case to occur in forwards time "would demand a vast number of distant coherent generators of waves" which would have to placed at "one centre" - but then of course this would not be the reverse of the usual case, and thus "this cannot be regarded as a possible classical process [18]." Price argues that Popper is guilty of a "temporal double standard", because towards the past waves converging to a centre are "exceedingly common [4]."

In a similar argument, Davies argues that waves are normally damped at the edges of a pond by friction, and the reverse case where particles randomly excite to produce a wave are "overwhelmingly improbable, though not impossible, on thermodynamic grounds [19] ."In other words, converging radiation would violate the second law of thermodynamics. However, as we have seen when using statistical reasoning to understand the second law, this argument would apply equally in the forward case, showing outgoing waves to also be improbable! As Price says, both cases "involve events in surrounding matter which are overwhelmingly unlikely on statistical grounds alone [4]." It seems that a thermodynamic argument alone is not sufficient to explain the radiative arrow of time.

4.2 Absorber theory

Waves and radiation are governed by Maxwell's equations, which are time symmetric. They allow for two types of solutions; retarded (outgoing waves), which leave a source and then arrive at a detector, and advanced (incoming waves), which arrive at a detector before leaving a source. The retarded solution describes that which we actually observe in nature, where as the advanced solution is never observed. Why is this the case? There are two possibilities. The first is that "there really is no incoming radiation in nature", in which case this is again a result of the nature of the early universe. The second possibility is that both kinds of radiation exist in nature, but for some reason we only observe incoming radiation when we look. Price favours the latter possibility, and argues its case with a modified version of Wheeler and Feynman's Absorber Theory of Radiation. Wheeler and Fenyman originally attempted to show the same thing as price, that radiation is symmetric and only looks like it isn't, but Price criticises this attempt, hence his modification [4].

The original Wheeler-Fenyman theory is as follows; consider an accelerated charge i emitting outgoing waves towards a surrounding absorber, as we observe it. This arrangement is illustrated in Figure 3:

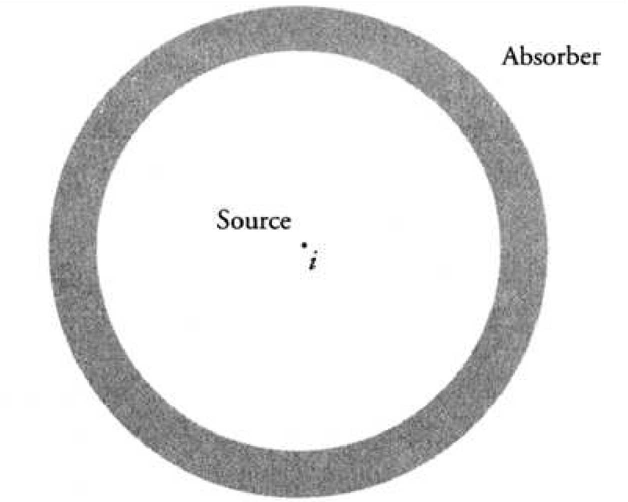


Figure 3: The Wheeler-Fenyman argument uses this arrangement [4]

Wheeler and Feynman suggest that half of the retarded wave we see as being emitted by the source i can be thought of as "the combined advanced waves of the absorber particles", with the other half being produced by i itself, under the assumption that "the initial contribution from the source is also half retarded and half advanced." The reason that we do not also observe an advanced wave at i is because the absorber particles produce a half-strength advanced wave exactly out of phase with the half strength wave from i, and so the two cancel out. So the source i contributes a radiation field which is 1/2 retarded + 1/2 advanced, and the absorber a field which is 1/2 retarded - 1/2 advanced, so that the field as a whole is a full value retarded wave [4].

Price's modification is to assume that "contrary to appearances", the incoming radiation “is coherently centred on the absorber particles.” It then follows, he argues, “that the same radiation field may be described equivalently either as a coherent wave front diverging from i, or as the sum of coherent wave fronts converging on the absorber particles.” Thus “both transmitters and receivers” are at the centre of concentric wave fronts. With this view things are symmetric at a microscopic level, with the existence of both microscopic coherent sources and sinks. However, at a macroscopic level we find only sources. Now what we need to explain is “why there are large coherent sources” in which “high number of tiny transmitters all act in unison”, but no equivalent large sinks. In other words, this argument leads to the conclusion that the radiative arrow of time has a cosmological origin, the same as the thermodynamic arrow of time. This is because if the universe was in thermal equilibrium microscopic receivers and transmitters would pop up as random fluctuations, but they would be uncorrelated an would not form larger sinks and sources. Since this is not the case, it must be that we see only large sources and not sinks because the universe is so far from thermodynamic equilibrium - which again takes us back to consider the Past State [4].

4.3 No explanation

As an alternative view, Frisch argues that “the puzzle of...radiative asymmetry...arises only if one subscribes to the view that the Maxwell equations on their own delineate the range of what is physically possible.” In other words, the reason that we do not see converging radiation is that whilst theory allows this, it is not actually possible in the real world. Frisch gives the example of solving quadratic equations as a similar case, saying “we often discard solutions given by the negative square root as unphysical without puzzling why ‘nature chooses’ only the positive root [20].” He also criticises Price’s modification of the Wheeler-Fenyman theory, questioning whether sources and sinks really are microscopically symmetric. He agrees that “both emitters and absorbers are centred on coherent wave fronts [4]” but argues that this is “an immediate consequence of the Maxwell equations and is not in need of support from the absorber theory or Price’s reinterpretation.” However this does not mean that “radiation is symmetric on the micro-level” since “there are emissions without absorptions, but no absorptions without re-emissions.” He also views Price’s modification as incorrect because it violates Gauss’s Law, one of Maxwell’s equations [20].

Frisch’s alternative is to question why an “additional constraint...beyond those given by the Maxwell equations” on what is possible is in need of an explanation. He asks if anyone would require an answer to why it is that not only do we only see fully retarded fields of radiation, but these fields also satisfy the Maxwell equations. Of course, no one (reasonable!) would. He adds “We do not find it puzzling that Gauss’s law alone does not determine the range of which is electrodynamically possible; the fact that there are four Maxwell equations...is not something we think is in need of an explanation. Frisch’s solution is, much like Callender proposed [15] for the cosmological arrow of time, a new law: “the retardation condition”, that “electromagnetic fields associated with a charge Q propagate along the future light cone of Q” (and only the future light cone) [20].

4.4 Summary

In this section I have examined the link between the radiative arrow of time with the two others, thermodynamic and cosmological. We have seen that the thermodynamic arrow does not explain the radiative arrow, but the cosmological arrow perhaps does. Price says “the radiative arrow becomes not so much a child of the second law as a junior sibling,” and I agree. Both arrows seem to owe their origin to that of the universe itself. For this reason I again reject the idea of lawhood - it must be that there is some explanation of the Past State which in turn explains both arrows.

5 Conclusion

Throughout this project we have seen that thinking about the arrow of time is no easy task. Before one even begins to question why time passes in the way it does, one must take a step back from the problem and consider what is being asked. In a nutshell, what we wish to discover is why physical processes take place only in one temporal direction, despite the fact that all of the scientific knowledge and equations we have to describe these processes do not specify this direction.

The first major work in this area was Boltzmann's H-theorem and subsequent ideas on thermodynamic entropy. Since then, many others have expanded on his work, leading to further ideas about the thermodynamic, cosmological and radiative arrows of time. From these ideas I think it is fair to say that links between the three arrows have been made, namely that the thermodynamic and radiative arrows are two separate phenomena, but they both have their origin in cosmology and the Past State.

Work on the thermodynamic arrow seems to me to be complete. The argument that a low-entropy state will always (given enough time) evolve to a high-entropy and eventually maximum-entropy state is a solid one, and does not require further investigation. Unfortunately, as we have seen, the argument also holds towards the past, and so is not a full explanation of the arrow of time. The radiative arrow, whilst perhaps on less firm ground than the thermodynamic arrow, also seems to be well understood whilst also not providing a full explanation.

It would seem that to explain the mystery of the arrow of time, "all" we have to do is explain the Past State. As mentioned in section 3.5, the leading candidates for explanation are a Gold universe, bestowing the low-entropy state on to both ends of the universe, or a future theory such as quantum gravity which would incorporate an asymmetric law that allows low-entropy initial singularities (i.e. the big bang) whilst forbidding low-entropy future singularities. I have come to the view that both theories are currently on equal footing, since there is neither more support for, nor evidence against, either one over the other. Hopefully one day new evidence will be discovered, or a new theory invented, that allows us to finally fully understand the arrow of time.

6 Appendix: Derivation of the H-theorem

The derivation of the H-theorem goes as follows [8]. First, some quantities must be defined. Boltzmann's equation for the evolution of a system of gas particles in the absence of any external force is (with repeated indices implying summation):

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} = \Omega(f) \tag{4}$$

We must make an assumption to define $\Omega(f)$, known as the collision function. This is the *stoßzahlansatz*, the "assumption of molecular chaos [4]", that the probability of velocities of particles are uncorrelated before a collision, but the same is not true after a collision. As we have seen, this is the reason for the theorem's failure in explaining the thermodynamic arrow of time, since it assumes time asymmetry to begin with. Continuing with the definitions, it means that $\Omega(f)$ is given by:

$$\Omega(f) = \int \int (f' f'_* - f f_*) g\sigma (\mathrm{ln} f + 1) d\Omega d\mathbf{v}_* d\mathbf{v} \tag{5}$$

Here $\mathbf{v}'$, $\mathbf{v}'_*$ are the velocities of two particles before a collision, $\mathbf{v}$, $\mathbf{v}_*$ are the velocities of the particles after a collision, and so $f' = f(\mathbf{x}, \mathbf{v}', t)$, $f'_* = f(\mathbf{x}, \mathbf{v}'_*, t)$, $f = f(\mathbf{x}, \mathbf{v}, t)$, $f_* = f(\mathbf{x}, \mathbf{v}_*, t)$. Also g is the magnitude of the particles relative velocity before collision, $d\Omega = \sin(\theta) d\theta d\phi$ is the solid angle (a sort of 3d analogue of the 2d angle [21]) the particles scatter into, σ is the differential cross section, and ϕ is the azimuthal angle in spherical coordinates. The exact nature of these quantities is not too important, as they are constant with respect to time. For collisions to conserve mass, momentum and energy, we also require:

$$\int \begin{bmatrix} 1 \\ \mathbf{v} \\ v^2 \end{bmatrix} \Omega(f) d\mathbf{v} = 0 \tag{6}$$

This is a compact way of defining what are known as the elementary collision invariants, $\varphi_i, i = 0, ..., 4$ where $\phi_0 = 1, \phi_1 = v_1, \phi_2 = v_2, \phi_3 = v_3, \phi_4 = v^2$, and $\int \varphi_i \Omega(f) d\mathbf{v} = 0$. It is also required that $\lim_{\mathbf{v} \to \infty}(f\varphi_i) = 0$ so that certain integrals converge. We can now continue with the derivation. Here is equation (1) again, for convenience:

$$H(t) = \int f(\mathbf{x}, \mathbf{v}, t) \mathrm{ln} f(\mathbf{x}, \mathbf{v}, t) d\mathbf{v} \tag{7}$$

Differentiating it with respect to time gives:

$$\frac{\partial H}{\partial t} = \int (1 + \mathrm{ln} f) \frac{\partial f}{\partial t} d\mathbf{v} \tag{8}$$

Substituting $\frac{\partial f}{\partial t}$ from equation (4) and $\Omega(f)$ from equation (5) yields:

$$\frac{\partial H}{\partial t} = -\underbrace{\int \left[\left(v_i \frac{\partial f}{\partial x_i} \right) (\mathrm{ln} f + 1) \right] d\mathbf{v}}_{1} + \underbrace{\int \int \int (f' f'_* - f f_*) g\sigma (\mathrm{ln} f + 1) d\Omega d\mathbf{v}_* d\mathbf{v}}_{2} \tag{9}$$

Consider part 1 of equation 9. Since $\mathbf{v}$ and $\mathbf{x}$ are independent, rewrite $v_i \frac{\partial f}{\partial x_i}$ as $\frac{\partial f}{\partial x_i} v_i$ and use the divergence theorem to get:

$$\int \left[\left(v_i \frac{\partial f}{\partial x_i} \right) (\mathrm{ln} f + 1) \right] d\mathbf{v} = \int (\mathrm{ln} f + 1) f \mathbf{v} \cdot d\mathbf{s} \tag{10}$$

where $d\mathbf{s}$ is the surface that encloses the volume $\mathrm{d}\mathbf{v}$ Since $\lim_{\mathbf{v} \to \infty}(f\varphi_i) = 0$, equation (10) vanishes, leaving equation (9) as

$$\frac{\partial H}{\partial t} = \int \int \int g\sigma (f' f'_* - f f_*) (\mathrm{ln} f + 1) d\Omega d\mathbf{v}_* d\mathbf{v} \tag{11}$$

For a reverse collision, in which particles with velocities $\mathbf{v}$ and $\mathbf{v}_*$ collide and move off with velocities $\mathbf{v}'$ and $\mathbf{v}'_*$, equation (11) becomes:

$$\frac{\partial H}{\partial t} = \int \int \int g\sigma (f' f'_* - f f_*) (\mathrm{ln} f_* + 1) d\Omega d\mathbf{v}_* d\mathbf{v} \tag{12}$$

The only term that has changed is $\mathrm{ln} f \to \mathrm{ln} f_*$ since $d\mathbf{v}_* d\mathbf{v} = d\mathbf{v}'_* d\mathbf{v}'$ as they are just dummy variables. Summing equations (11) and (12) then dividing by two gives:

$$\frac{\partial H}{\partial t} = \frac{1}{2} \int \int \int g\sigma (f' f'_* - f f_*) (\mathrm{ln} f + \mathrm{ln} f_* + 2) d\Omega d\mathbf{v}_* d\mathbf{v} \tag{13}$$

Swapping the dummy variables $\mathbf{v} \leftrightarrow \mathbf{v}'$ and $\mathbf{v}_* \leftrightarrow \mathbf{v}'_*$ yields:

$$\frac{\partial H}{\partial t} = \frac{1}{2}\int\int\int g\sigma(f'f'_* - ff_*)(-\ln f - \ln f_* + 2)d\Omega d\mathbf{v}_* d\mathbf{v} \tag{14}$$

Now finally summing equations (13) and (14) and dividing by 2 gives:

$$\frac{\partial H}{\partial t} = \frac{1}{4}\int\int\int g\sigma(f'f'_* - ff_*)\ln\left(\frac{ff_*}{f'f'_*}\right)d\Omega d\mathbf{v}_* d\mathbf{v} \tag{15}$$

Now clearly $(f'f'_* - ff_*)$ and $\ln\left(\frac{ff_*}{f'f'_*}\right)$ take opposite signs and everything else in equation (15) is always positive, and thus:

$$\frac{\partial H}{\partial t} \leq 0 \tag{16}$$

This proves that H never increases. It can also be shown that H tends to a finite limit corresponding to $\frac{\partial H}{\partial t} = 0$, and this corresponds to the Maxwell distribution [8].

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